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Reducing the variability of inter-departure times of a single-server queueing system – The effects of skewness

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ABSTRACT

A critical performance measure in serial production lines is the variability of inter-departure times of a single-server queue. Increasing upstream inter-departure time variability generates greater downstream variability, diminishing overall line performance. Theory suggests that the variability of inter-departure times of a single-server queue is reduced by decreasing the variance of inter-arrival and service times. This study investigates the effects of the skewness of inter-arrival and service time distributions on the variability of inter-departure times. Contrary to previous results suggesting that mean waiting times of a GI/G/1 queue can be reduced by increasing inter-arrival time skewness, this experimental study of a GI/G/1 queue with triangular inter-arrival and service times shows that the inter-departure time coefficient of variation is reduced through a combination of negative inter-arrival time skewness and positive service time skewness. These results also suggest that the absolute value of the negative autocorrelation between consecutive departures is reduced by the same combination of negative inter-arrival time skewness and positive service time skewness for low values of server's utilization, while positive skewness for both inter-arrival and service times reduces this value for high values of server's utilization. Finally, it was found that queue capacity constraints increase the coefficient of variation of inter-departure times, as has been previously suggested, as well as the skewness and the absolute correlation values of the inter-departure time distribution.

1. Introduction

In today's hyper-competitive business world, advantage can result from accruing small improvements. As in sporting events, progressive process improvements of seconds, or even milliseconds, can result in stark differences between competitors, lower costs, and increased profit. When examining the management of production lines and serial supply systems, a focus on the variability of inter-departure times of single-stage queueing systems is a valuable research topic, since variability can undermine performance expectations on a serial line.

Typically, the probability distribution of inter-departure times is the factor linking the stations of a serial line, i.e. the inter-departure time distribution of an upstream station is the inter-arrival time distribution of a downstream station. Consequently, an increased variability of inter-departure times for an upstream station leads to a propagation of variability for downstream stations, and potential complications. Higher variability of inter-arrival times for downstream stations increases their mean waiting time, negatively affecting the performance of the entire serial line. For these reasons, a comprehensive understanding of the causes of variability of inter-departure times of individual stations is desired, particularly to fill the gaps that exist in current knowledge.

Classic observations regarding inter-departure times produce some straightforward conclusions. For example, for any single-stage queueing system with a utilization factor of less than 1, the mean inter-departure time is equal to the mean inter-arrival time, a conclusion that extends to simple serial lines. Another example, based on the results of an M/M/s system, indicates the Poisson arrival process is the same as the departure process (Burke, 1956). But for GI/G/1 and GI/G/s systems, i.e. general distributions of independent inter-arrival times and general distributions of service times, the departure process is not equal to the arrival process (Daley, 1976).

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Furthermore, several authors (Buzacott & Shanthikumar, 1993; Hopp & Spearman, 2000; Tripathi & Agrawala, 1982) have suggested that the variability of inter-departure times, as measured by its coefficient of variation (CV), is not equal to the CV of inter-arrival times. They suggested that the CV of inter-departure times is the result of the interaction among the server's utilization factor, inter-arrival time CV and service time CV. But none of these studies have specifically considered how higher moments of the inter-arrival and service time distributions actually influence the variability of inter-departure times.

Complementary research by Klincewicz and Whitt (1984) and Johnson and Taaffe (1991) showed that the skewness of inter-arrival times has a significant effect on the mean queue length considering the GI/M/1 queue. Sahin and Perrakis (1976) and Whitt (1984) also suggested that inter-arrival time skewness has a greater influence on the mean queue length than does service time skewness in GI/G/1 queues and that a positive inter-arrival skewness reduces the mean waiting time of the queue.

These results provide a solid base which raises two relevant research questions on the performance of GI/G/1 queues:

1. What effects do the skewness of inter-arrival times and the skewness of service times have on the stochastic behavior of inter-departure times?
2. Can the variability of inter-departure times be reduced only by modifying the values of the skewness of the distributions of inter-arrival and service times as their CVs and the utilization of the server remains constant?

This paper will vary the skewness of inter-arrival and service time distributions to examine their effects on the distribution of inter-departure times of a single-server queue. To comprehensively investigate, an experimental simulation study of a GI/G/1 system with triangular inter-arrival and service time distributions was developed and conducted.

The paper is organized as follows. Section 2 presents a literature review. The study's methodological approach is presented in Section 3, while Section 4 reports the results of the investigation. Finally, Sections 5 and 6 present the discussion and conclusions, respectively.

2. Literature review

The output process of a single-server queue has been widely studied because of its relevance to the Production Management and Operations Research fields. Studies have presented analytical solutions of the output process of various M/G/1, M/G/s and G/M/1 queueing systems (Chaudhry, Agarwal, & Templeton, 1992; Daley & Shanbhag, 1975; Finch, 1959; King, 1971; Nazarathy & Weiss, 2008; Stanford, 1991; Veeger, Kerner, Etman, & Adan, 2011; Yeh & Chang, 2000) and a number of conclusions can be drawn from these studies.

For example, Finch (1959) showed that two successive departure intervals are independent. Yeh and Chang (2000) suggested that the dependency among inter-departure times is reduced by the variance of the service time distribution and is increased by the system's load. Nazarathy and Weiss (2008) found that when the utilization factor of a M/M/1/k system equals one (BRAVO effect = Balancing Reduces Asymptotic Variance of Outputs), the asymptotic variance of the output process is reduced. This conclusion was extended to the M/M/1 system by Al-Hanbali, Mandjes, Nazarathy, and Whitt (2011) and was used as the basis for Hautphenne, Kerner, Nazarathy, and Taylor (2015) to propose a procedure to calculate the variance rate and intercept terms for the asymptotic variance curve of M/M/1/k and M/G/1 queues.

Despite the value of that research stream, the departure process of a GI/G/1 system has received less attention and has been less frequently examined because its general analytical results are more difficult to obtain. For example, Vlach and Disney (1969) presented a fairly complex procedure that models the inter-departure time distribution as a two-dimensional Markov Chain composed of states depending on queue size at the nth departure and on the time since the last arrival, measured from the instant of the nth departure. They concluded that a utilization factor of less than 1 is a sufficient condition for the output process to possess a stationary distribution. Jain and Grassmann (1988) presented a method to calculate the first two moments of the distribution of inter-departure times of a G/G/1 system that depends on the calculation of mean idle time, variance of idle times and probability that an arrival has to wait.

Hu (1996) and Girish and Hu (2001) suggested using recursive equations, based on the work of Lindley (1952), to calculate the first three moments of the inter-departure time distribution of a G/G/1 system with Markov-modulated arrivals. To prove the validity of the method, Girish & Hu proposed an equation to calculate the variance of an M/G/1 queue. They suggested that each moment of the inter-departure time distribution can be calculated via the lower moments of the distributions of inter-arrival, service, cycle and waiting times.

Following Girish and Hu (2001), Kumaran, Mitchel, and van de Liefvoort (2004) presented an approximation technique to model the departure process of a G/G/1 queue by modelling the arrival and service processes as matrix exponential processes after fitting such processes to a phase-type distribution (Gerhardt & Nelson, 2010). Because of their analytical approach, they were also able to approximate any kth-moment of the departure process. They also showed that, for a H2/E10/1 queue, the autocorrelation of the departure stream diminished as the coefficient of variation of the arrival process increased and that the squared coefficient of variation of the departure stream diminished as the utilization of the server increased.

Buzacott and Shanthikumar (1993) and Hopp and Spearman (2000) similarly suggested general formulas to find the variability of inter-departure times by estimating the inter-departure time squared coefficient of variation (CV^2) of a GI/G/1 system. Hopp & Spearman stated that these formulas could be used as a 'linking formula' for serial production lines to link the stochastic output processes of upstream single stations with the input process of downstream stations.

In 1993, Buzacott & Shanthikumar presented three formulas for approximating the CV^2 of inter-departure times. Because of their relevance to the current study, these approximations are shown below and followed by Hopp & Spearman's approximation formula for computing the CV^2 of inter-departure times for a single-server queue:

$$(1-\rho^2)\left(\frac{CV_a^2+\rho^2 CV_s^2}{1+\rho^2 CV_s^2}\right)+\rho^2 CV_s^2 \tag{1}$$

$$1-\rho^2+\rho^2 CV_s^2+(CV_a^2-1)\left(\frac{(1-\rho^2)(2-\rho)+\rho CV_s^2(1-\rho)^2}{2-\rho+\rho CV_s^2}\right) \tag{2}$$

$$(1-\rho)(1+\rho CV_a^2)CV_a^2+\rho^2 CV_s^2 \tag{3}$$

where

ρ is the utilization factor of the server,
CV_a is the coefficient of variation of inter-arrival times and,
CV_s is the coefficient of variation of service times.

$$\rho^2 CV_s^2+(1-\rho^2)CV_a^2 \tag{4}$$

The above formulas describe the heavy-traffic asymptotic behavior of the variability of inter-departure times, since the CV^2 of inter-departure times is limited by the CV^2 of service times when the utilization factor tends to reach 1. Similarly, as the utilization factor approaches zero, the CV^2 of inter-departure times is significantly influenced by the CV^2 of inter-arrival times.

Several pertinent studies have specifically addressed the impact of the skewness on the performance of single server queues. For example, Whitt (1984) shows that the impact of inter-arrival time skewness on mean queue length is much higher than the impact of service time skewness, making negligible the actual impact of service time skewness for GI/G/1 queues with phase-type distributions. Johnson and Taaffe (1991) later concluded that when the CV of inter-arrival times is low, the impact of the skewness on mean queue length is low and vice-versa, for a G/M/1 queue. They also observed that the impact of any of the first three moments of the inter-arrival time distribution is reduced when the utilization of the server increases.

More recently, Wu, Srivathsan, and Shen (2018) proposed a three-moment approximation formula for GI/G/1 queues, suggesting that a two-moment approximation should be used when the CV^2 of inter-arrival times is less than one. Otherwise, they suggest using a three-moment approximation.

Despite previous research concluding that the variability of inter-departure times (variance and CV) of a GI/G/1 queue is influenced by the interaction of the server's utilization factor and the mean and variance of inter-arrival and service times, and previous research concerns with analytically characterizing the exact distribution of inter-departure times (Ferng & Chang, 2001; Shioda, 2003; Zhang, Heindl, & Smirni, 2005; Zhang, Heindl, Smirni, & Stathopoulos, 2009), no specific indication has yet been found regarding how the departure process is influenced specifically by the skewness of inter-arrival and service time distributions. This study intends to fill that gap in the theoretical literature and contribute to greater queueing theory understanding.

3. Methodology

Previous analytical approaches have not considered the impact of the skewness of inter-arrival and service time distributions due to the extreme difficulty of obtaining exact expressions for non-phase-type distributions, including some GI/G/1 systems (Righter, 2002; Wolff, 1970). Although several methodological options exist in the literature to overcome the expression constraints and challenges, an experimental simulation approach can also be applied to an exploratory research setting to determine the behavior of the system concerning the effect of inter-arrival and service skewness on the variability of inter-departure times.

Discrete-event simulation is an effective modelling tool to deal with stochastic systems, such as the GI/G/1 queue. It allows the researcher to conduct a variety of experiments by manipulating the system's parameters to study the effect of certain variables on the system, and is well-suited to the setting (Gue & Kim, 2012; Lagershausen & Tan, 2015). Thus, discrete-event simulation was deemed an appropriate investigative methodology for this study.

The triangular distribution was selected as a probability distribution to model inter-arrival and service times of the system. The triangular distribution is a probability function that can take on different values of skewness, i.e. positive and negative values, while maintaining the same fixed values of the mean and variance. This characteristic is critical in the experimental design of this study, as the effect of contrasting skewness values on the inter-departure time distribution needed to be studied without considering the impacts of the mean and variance of inter-arrival and service times. Furthermore, the triangular distribution can be used as a rough model of random variates (Law, 2014) with a low value of CV, a characteristic that has been found present in real production scenarios (Doerr, Freed, Mitchell, Schriesheim, & Zhou, 2004; Inman, 1999). Therefore, each experiment was designed with equal values of both inter-arrival and service time CV. This factor was termed the 'system's CV', which represents both the CV of inter-arrival and service times.

Five factors and five responses were considered in the experimental design. The factors and their levels are shown in Table 1. Since the skewness factor is most pertinent in this study, three experimental levels for the skewness of inter-arrival and service time distributions were included and multiple levels for the server's utilization were selected, while only two levels for the system's CV were considered. In addition, since queue (buffer) capacity has been shown to influence the output process of serial lines (Hendricks & McClain, 1993; Kalir & Sarin, 2009) and real queueing systems could have limited queue capacity, three levels of queue capacity were considered. (See Table 2).

Table 1
Experimental factors and their levels.

Factor	Factor levels: −1	0	+1
Skewness of inter-arrival times (γA)	−0.42 or −0.57	0	0.42
System's CV	0.2318		0.3642
Server's utilization (ρ)	0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.85, 0.90, 0.95, 0.97, 0.98, 0.99		
Skewness of service times (γS)	−0.42 or −0.57	0	0.42 or 0.57
Queue capacity (B)	8	64	∞

Table 2
ANOVA test for CV_{ID}, differentiated by experimental CV.

	CV = 0.2318			CV = 0.3642		
	Sum Sq	F value	Pr(> F)	Sum Sq	F value	Pr(> F)
γ_A	0.142	311.228	0.000	0.521	1311.134	0.000
γ_S	0.035	76.376	0.000	0.037	92.753	0.000
ρ	0.466	1022.922	0.000	0.008	18.890	0.000
B	0.000	0.217	0.805	0.004	5.215	0.005
γ_A:γ_S	0.000	0.005	0.943	0.000	0.023	0.880
γ_A:ρ	0.013	29.463	0.000	0.015	37.272	0.000
γ_S:ρ	0.012	26.515	0.000	0.007	17.686	0.000
γ_A:B	0.000	0.001	0.999	0.000	0.026	0.974
γ_S:B	0.000	0.001	0.999	0.000	0.021	0.979
ρ:B	0.000	0.257	0.773	0.005	5.954	0.003
γ_A:γ_S:ρ	0.000	0.108	0.742	0.000	0.498	0.480
γ_A:γ_S:B	0.000	0.000	1.000	0.000	0.001	0.999
γ_A:ρ:B	0.000	0.001	0.999	0.000	0.029	0.971
γ_S:ρ:B	0.000	0.001	0.999	0.000	0.025	0.976
γ_A:γ_S:ρ:B	0.000	0.000	1.000	0.000	0.001	0.999
Residuals	17.192			15.019		

The initial values for the triangular distributions were taken from Khalil, Stockton, and Fresco (2008). The resulting levels of the system's CV (i.e. 0.2318 and 0.3642) were generated from those initial values. Furthermore, two practical single-server queues were selected to represent realistic queue capacities, while a theoretical infinite capacity was also considered. Taking inspiration from Bhatnagar, Patel, and Karmeshu (2018) FTP packet-processing router model, a queue capacity of 64 was selected; whereas a commercially available telephone PBX system (Costa, Nunes, Bordim, & Nakano, 2015) with a capacity for holding 8 external calls was taken as an example of a more constrained queue capacity.

A total of 756 experiments resulted from this design. Moreover, 84 groups of 9 experiments were built, each with the same CV, ρ and B values but with different γ_A and γ_S values. This experimental design allowed an exclusive and comprehensive study on the effect of skewness under various traffic intensities and queue capacities.

It has been previously shown that the skewness (Johnson, 1993; Whitt, 1984) and Lag-1 autocorrelation (Livny, Melamed, & Tsiolis, 1993) of the arrival process (inter-arrival times) have a significant effect on the performance of a single-server queue. Considering that the departure process (inter-departure times) of an upstream queue could be the arrival process of downstream queue, it is important to not only investigate the effects of inter-arrival and service skewness on the variance of inter-departure times but also their effects on the skewness and Lag-1 autocorrelation of inter-departure times.

Therefore, the experimental setting of this study considered the following responses per experiment:

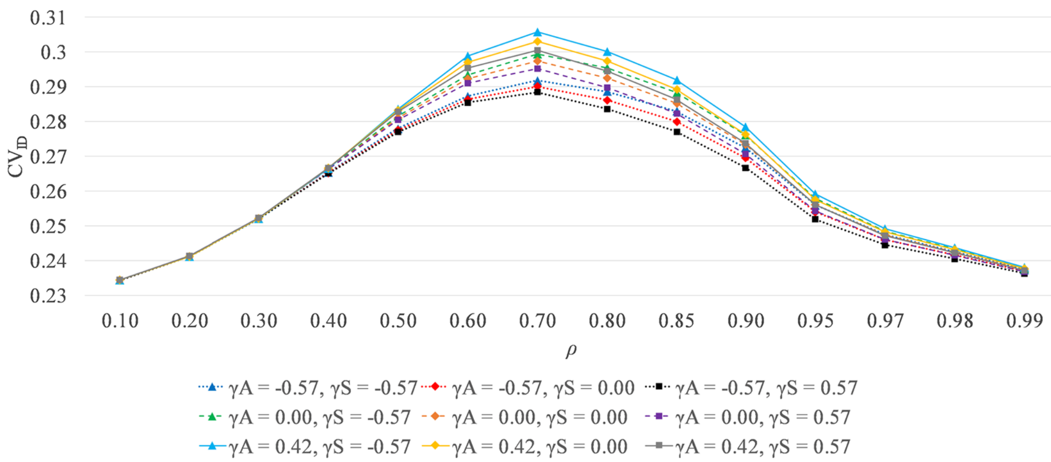

Fig. 1. CV_{ID} values for experiments with a system's CV = 0.2318.

– the mean inter-departure time (μ_{ID}, which should be equal to the mean inter-arrival time)

$$\mu_{IDj} = \frac{1}{n}\sum_{i=1}^{n} X_{ij} \tag{5}$$

where X_{ij} is the ith inter-departure time between the $i + 1$th departure and the ith departure of the jth replication

– the variance of inter-departure times (σ^2_{ID})

$$\sigma^2{}_{IDj} = \frac{1}{n-1}\sum_{i=1}^{n} (X_{ij} - \mu_{IDj})^2 \tag{6}$$

– the coefficient of variation of inter-departure times (CV_{ID})

$$CV_{IDj} = \sigma_{IDj}/\mu_{IDj} \tag{7}$$

– the skewness of the distribution of inter-departure times (γ_{ID})

$$\gamma_{IDj} = \frac{\frac{1}{n}\sum_{i=1}^{n}(X_{ij} - \mu_{IDj})^3}{\left[\frac{1}{n-1}\sum_{i=1}^{n}(X_{ij} - \mu_{IDj})^2\right]^{3/2}} \tag{8}$$

– the Lag-1 autocorrelation of consecutive inter-departures ($Lag1_{ID}$)

$$Lag1_{IDj} \approx \frac{\sum_{i=1}^{n-1}(X_{ij} - \mu_{IDj})(X_{(i+1)j} - \mu_{IDj})}{\sigma^2{}_{IDj}} \tag{9}$$

A total of 100 independent simulation runs (N = 100) of n customers were carried out for each experiment with a warm-up period of w customers, using Simio 10.165 simulation software (Kelton, Smith, & Sturrock, 2014). The values of n and w were estimated using Welch's method (Welch, 1983) and depended on the value of ρ (refer to Table A1 in the Appendix for the actual values of n and w and to Fig. A1 for an example of one of the Welch's method results). This resulted in a total of 100 results per each of the five responses. The results reported in this paper correspond to the average of the 100 replication results per response. For instance, the variance response of inter-departure times for each experiment was calculated as:

$$\sigma^2{}_{ID} = \frac{1}{N}\sum_{j=1}^{N}\sigma^2{}_{IDj} \tag{10}$$

In addition, the significance of the differences among the results of different experiments within the same group (experiments with the same CV, ρ and B values but different γ_A and γ_S) was calculated to assess the effect of the skewness on the responses. Statistically significant differences among experiments with same CV, ρ, γ_A and γ_S values but different B values, were also calculated to investigate the effect of queue capacity on each response. Firstly, ANOVA tests (Walpole, Myers, Myers, & Ye, 2011) were conducted to see if a statistical effect on each response was caused by modifying the values of γ_A and γ_S. Secondly, Duncan's multiple range tests (Duncan, 1955) were performed to each group of experiments to analyze whether statistically significant differences exist among the averages of the responses of all experiments within the group. Finally, additional ANOVA tests describing each response were conducted to assess the statistical significance of the main effects and interactions on each response. All statistical calculations were performed using R 3.4.0 (R Foundation, 2016).

It is worth noting that, as some of the experimental values in this study consider a finite queue capacity, customer balking (a customer leaving the system immediately after arrival – Ward & Glynn, 2005), occurred whenever a customer arrived into the system and found a full queue. For example, for B = 8, a customer balked if and only if there were already 9 customers in the system: one being serviced and 8 more waiting in the queue.

4. Results

According to previous approximations (Buzacott & Shanthikumar, 1993; Hopp & Spearman, 2000; Rao & Feldman, 1999), the results for μ_{ID}, σ^2_{ID} and CV_{ID} should be equivalent for all experiments pertaining to the same group (same system's CV and ρ). As expected, μ_{ID} for all within-group experiments with infinite queue capacity resulted in very similar values with no statistically significant differences.

The results regarding CV_{ID}, γ_{ID} and $Lag1_{ID}$ are presented in the next subsections as the results for σ^2_{ID} and CV_{ID} are equivalent for this set of experiments.

4.1. Coefficient of variation of inter-departure times

Results for the coefficient of variation of inter-departure times (CV_{ID}) varied among experiments included in the same group, as shown in Figs. 1 and 2. A previously unseen and unreported pattern for inter-departure times can be seen in the results obtained for CV_{ID}, as a positive value of γ_A resulted in higher values for CV_{ID}, an opposite result from the one pertaining to mean waiting times, since a positive γ_A translates into lower mean waiting times (Johnson, 1993; Whitt, 1984). Conversely, a positive value of γ_S resulted in smaller values for CV_{ID}.

Furthermore, a quadratic behavior of CV_{ID} depending on ρ can be clearly seen in Figs. 1 and 2 (the graphical 'hump'), since experiments with low and high ρ values have very similar values to the original CV value of inter-arrival and service times, while experiments with intermediate ρ values have higher CV_{ID} than the system's CV.

It is worth noting that CV_{ID} results for experiments with

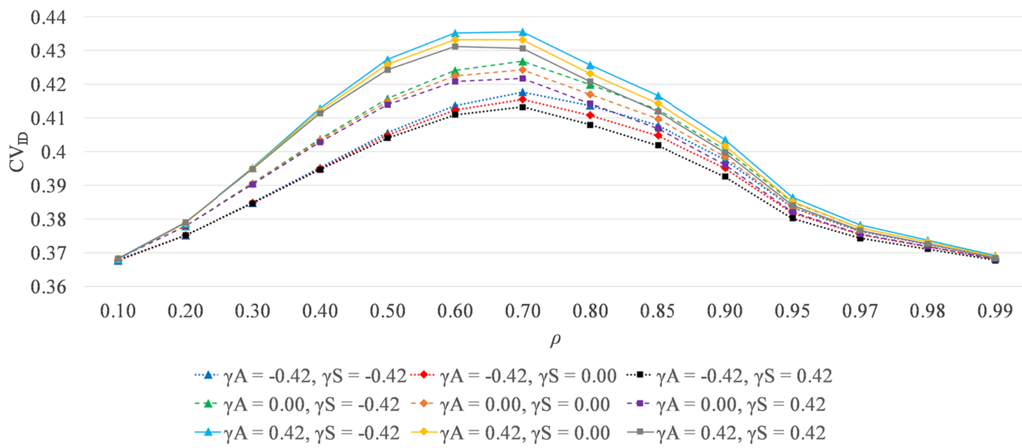

Fig. 2. CV_{ID} values for experiments with a system's CV = 0.3642.

CV = 0.2318 and ρ equal to 0.1, 0.2 and 0.3 did not have statistically significant differences among experiments in the same group, as seen in Table A2 in the Appendix.

The quadratic behavior of CV_{ID} depending on ρ shown in Figs. 1 and 2 is better characterized by the approximation formula suggested by Buzacott and Shanthikumar (1993) (1) for estimating CV_{ID}, when compared with formulas (2), (3) and (4), as shown in Fig. 3. Similarly, the effects of γ_A and γ_S on CV_{ID} were highly influenced by ρ, as the differences between experiments within the same group (same values of ρ and CV) were higher when $0.5 \le \rho \le 0.9$ (see Tables A2 and A3 in the Appendix); whereas CV_{ID} was more dependent on γ_A for lower ρ and seemingly equally dependent on both γ_A and γ_S for the highest values of ρ.

On the other hand, Fig. 4 suggests that the effects of γ_A and γ_S on CV_{ID} are fairly similar for different B values, i.e. decreasing γ_A values and increasing γ_S values reduce CV_{ID}. Fig. 4 also confirms the results from previous studies (Hendricks & McClain, 1993; Kalir & Sarin, 2009) showing that lower B results in higher σ^2_{ID} and, as a consequence, higher CV_{ID}.

Fig. 4 also suggests that the influence of B on CV_{ID} was more important in very high traffic ($\rho \ge 0.98$) than the effects of γ_A and γ_S. This was caused by the fact that, for experiments with B = 8, balking occurred only when $\rho \ge 0.85$ (see Tables A14 and A15 in the Appendix); thus, statistically significant differences in terms of CV_{ID} among experiments with different B values were only present in the higher values of ρ, as shown in Tables A4 and A5 in the Appendix. Furthermore, since the probability of balking when B = 64 was different from zero only when $\rho \ge 0.98$ (Tables A14 and A15 in the Appendix), no statistically significant differences were found among experiments with B = 64 and $B = \infty$ in terms of CV_{ID} (Tables A4 and A5 in the Appendix), γ_{ID} (Tables A8 and A9 in the Appendix) and $Lag1_{ID}$ (Tables A12 and A13 in the Appendix).

An assessment of the actual impact of each factor on the resulting experimental CV_{ID} is reflected in Table 3, which shows the ANOVA test of CV_{ID}, where it can be seen that ρ has the highest impact on CV_{ID}, followed by the impact of γ_A for experiments with CV = 0.2318; whereas for experiments with CV = 0.3642, γ_A is the most impactful factor in terms of CV_{ID}. Interestingly, the effect of γ_S was also statistically significant, albeit less strong, and also the interaction effects between ρ and γ_A as well as ρ and γ_S; while the effect of B was, surprisingly, not significant for experiments with CV = 0.2318. The lack of interaction between B and both γ_A and γ_S is also shown by the statistically non-significant relationship between these variables, i.e. γ_A:B and γ_S:B.

4.2. Skewness of inter-departure times

The study of the impact of γ_A and γ_S on the skewness of inter-departure times (γ_{ID}) is a relevant research topic. Since the output process of an upstream queue will be the input process of a downstream queue and positive γ_A will reduce mean waiting times, it can increase the performance of the queue.

In this regard, Figs. 5 and 6 suggest that γ_{ID} is highly dependent on γ_A at lower ρ values and highly dependent on γ_S at higher ρ values. This particularity can be clearly seen in the groups formed by the Duncan's test (Tables A4 and A5 in the Appendix) which shows that experiments with the same values for γ_A have no statistically significant differences at lower ρ, and form the same groups in the test. Moreover, the highest values of γ_{ID} are reached in intermediate values of ρ while the lowest values of γ_{ID} are reached on very low or very high values of ρ. An

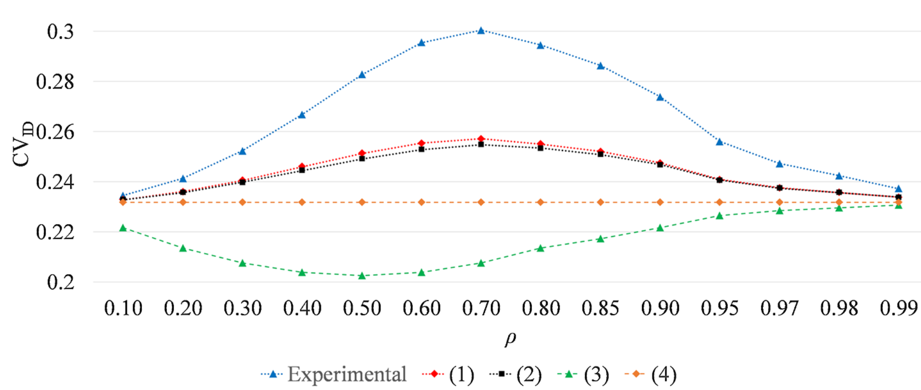

Fig. 3. Comparison of formulas to estimate CV_{ID} against experimental results for different values of ρ and CV = 0.2318, $\gamma_A = 0.42$ and $\gamma_S = 0.57$.

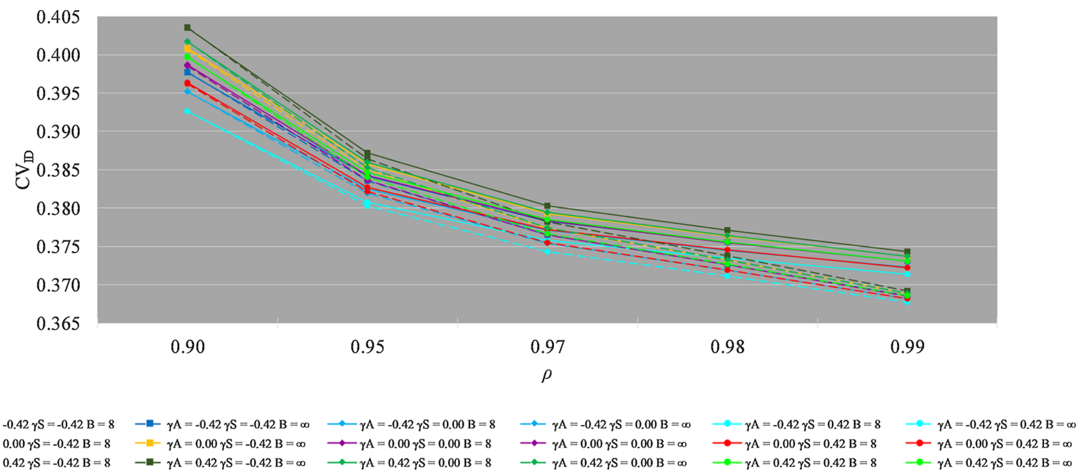

Fig. 4. Effect of γ_A and γ_S on CV_{ID} for different values of B for $\rho \geq 0.90$ and CV = 0.3642.

Table 3
ANOVA test for γ_{ID}.

	Sum Sq	F value	Pr(> F)
γ_A	1183.000	43305.406	0.000
γ_S	1078.500	39481.049	0.000
ρ	1399.600	51231.719	0.000
CV	54.800	2004.423	0.000
B	0.900	16.265	0.000
γ_A:γ_S	0.000	1.253	0.263
γ_A:ρ	1004.800	36779.878	0.000
γ_S:ρ	868.100	31778.371	0.000
γ_A:CV	5.000	181.573	0.000
γ_S:CV	8.700	317.249	0.000
ρ:CV	35.900	1313.222	0.000
γ_A:B	0.000	0.204	0.815
γ_S:B	0.100	2.075	0.126
ρ:B	1.000	18.840	0.000
CV:B	0.300	5.949	0.003
γ_A:γ_S:ρ	0.000	0.591	0.442
γ_A:γ_S:CV	0.000	0.011	0.917
γ_A:ρ:CV	2.200	80.415	0.000
γ_S:ρ:CV	3.400	125.845	0.000
γ_A:γ_S:B	0.000	0.002	0.998
γ_A:ρ:B	0.000	0.238	0.788
γ_S:ρ:B	0.100	2.409	0.090
γ_A:CV:B	0.000	0.036	0.965
γ_S:CV:B	0.100	1.052	0.349
ρ:CV:B	0.400	6.750	0.001
γ_A:γ_S:ρ:CV	0.000	0.099	0.753
γ_A:γ_S:ρ:B	0.000	0.002	0.998
γ_A:γ_S:CV:B	0.000	0.000	1.000
γ_A:ρ:CV:B	0.000	0.041	0.959
γ_S:ρ:CV:B	0.100	1.194	0.303
γ_A:γ_S:ρ:CV:B	0.000	0.000	1.000
Residuals	2063.900		

exception exists in experiments with a positive γ_A, which seem to exhibit cubic behavior dependent on ρ, a characteristic that can be seen in the 'valley' between $0.2 \leq \rho \leq 0.6$ in Figs. 5 and 6.

The experimental γ_{ID} values appear to be the result of a weighted average between γ_A and γ_S values, with a graphical hump for intermediate values of ρ. This behavior is considered by approximation formulas (1), (2), (3) and (4). Hence, these formulas were used to approximate γ_{ID} by substituting the CV_A^2 and CV_S^2 terms by the terms γ_A and γ_S, respectively, to assess if these formulas could be useful to estimate γ_{ID}. The result of this exercise for two contrasting values of γ_A and γ_S is presented in Fig. 7, where the approximation formulas exhibit good performance in extreme ρ values but poorly model the 'hump' behavior caused by intermediate values of ρ, especially considering a positive value for γ_A (Fig. 7b).

A final issue regarding γ_{ID} is the influence of each factor on experimental γ_{ID}. Table 3 shows that the factor with the highest effect on γ_{ID} was ρ, followed by γ_A and γ_S. As seen also in Figs. 5 and 6, the interactions between ρ and γ_A and ρ and γ_S are essential for the results of γ_{ID}, while the effect of CV, although statistically significant, is not large.

The small but statistically significant influence of B on γ_{ID} is shown in Fig. 8, where it can be seen that B has a relevant impact only when queue capacity constraints result in a significant proportion of customers balking, i.e. the difference between dashed (unlimited capacity) and solid lines (limited queue capacity) is more apparent when ρ approaches 1. Moreover, Fig. 8 and Table 3 also suggest that the relationship between increasing (or decreasing) values of both γ_A and γ_S on γ_{ID} does not change with changing values of B and that imposing a limit on B, e.g. when B = 8 (solid lines), will result in higher skewness.

4.3. Lag-1 autocorrelation of inter-departure times

Livny et al. (1993) showed that either positive or negative autocorrelation between arrivals could significantly affect the performance of a single-server queue. Thus, the Lag-1 autocorrelation coefficient of inter-departure times ($Lag1_{ID}$) was also investigated in this experimental setting.

Figs. 9 and 10 show an 'inverse' graphical hump regarding the values of $Lag1_{ID}$. Similar to the results of γ_{ID}, results for $Lag1_{ID}$ seem to be more dependent on γ_A at low values of ρ and more dependent on γ_S at high ρ values. However, the comparative effect of different γ_A values changes with increasing ρ. Negative γ_A at low ρ values resulted in lower absolute magnitudes (less negative values) of $Lag1_{ID}$, whereas negative γ_A at high ρ values resulted in higher absolute magnitudes (more negative values) of $Lag1_{ID}$, when compared with experiments with other γ_A values.

No statistically significant differences were found among experiments with very low values of ρ, as shown in Tables A10 and A11 in the Appendix. Tables A10 and A11 also illustrate how, at low ρ values ($0.2 \leq \rho \leq 0.4$) for experiments with CV = 0.3642, the groups formed by Duncan's test tend to be formed by experiments with the same γ_A. However, for higher ρ, experiments with different γ_A and γ_S values did have statistically significant differences.

Table 4 results suggest that γ_S and the interaction between ρ and γ_S had a much higher effect on $Lag1_{ID}$ than γ_A, although the highest effect on $Lag1_{ID}$ was caused by the CV of inter-arrival and service times. The effect of CV is clearly shown in the difference between Figs. 9 and 10 as $Lag1_{ID}$ values for the experiments with CV = 0.2318 had a higher absolute magnitude (more negative values) than $Lag1_{ID}$ values for experiments with CV = 0.3642; whereas the effect of B on $Lag1_{ID}$,

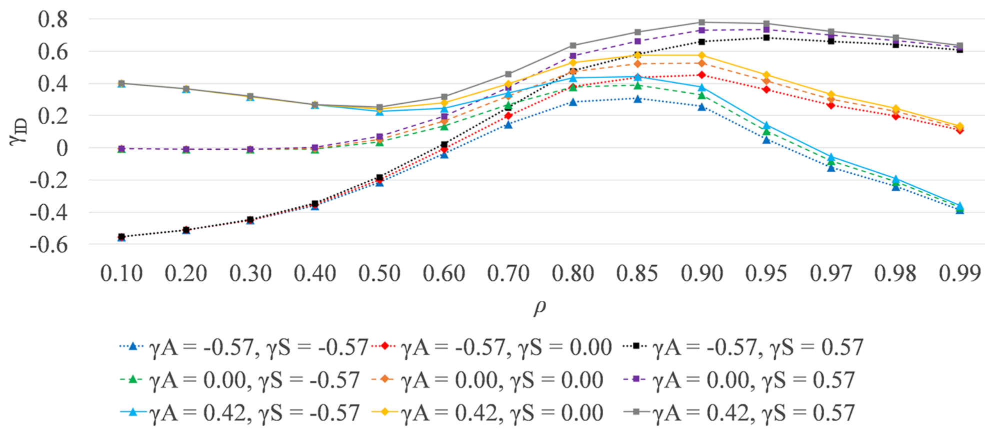

Fig. 5. γ_{ID} values for experiments with a system's CV = 0.2318.

although significant, was not large.

The influence of B on $Lag1_{ID}$ (see Fig. 11) was similar to its influence on CV_{ID} as a limit on B created more variability in the queue, in this case, more negative $Lag1_{ID}$ values. Also, as was the case with CV_{ID}, the higher ρ was, the higher the influence of B on $Lag1_{ID}$ was, due to the impact of B on the probability of balking for very high traffic scenarios. Regardless, results shown in Fig. 11 suggest that the effect of γ_A and γ_S on $Lag1_{ID}$ was not influenced by B (also shown by the lack of statistical significance of γ_A:B and γ_S:B interactions).

5. Discussion

Conventional queueing theory suggests that the variability of inter-departure times is affected by the mean and variance of inter-arrival and service time distributions. This investigation into the effects of the skewness of triangular inter-arrival and service times on the variability of inter-departure times was motivated by previous work that showed the skewness of inter-arrival times has a significant effect on mean queue length, and to contribute to a better understanding of performance causes in single-server queues and serial lines.

Study results show that the skewness of inter-arrival and service time distributions do influence CV_{ID}, although ρ and CV of inter-arrival and service times remain key factors for determining CV_{ID}. Positive skewness of inter-arrival times created higher CV_{ID} while positive skewness of service times resulted in lower CV_{ID} throughout the complete range of experimental values of CV, ρ and B, directly answering research question 1.

This result, leading to research question 2, suggests that CV_{ID} could be reduced by implementing a combination of negative inter-arrival skewness and positive service skewness. This differs from previous conclusions (Johnson, 1993; Romero-Silva, Shaaban, Marsillac, & Hurtado, 2017; Whitt, 1984) suggesting that the waiting times of a GI/G/1 queue increased by decreasing inter-arrival skewness and increasing service skewness for a system with CV < 1, similar to the results from the current study (see Tables A14 and A15 in the Appendix) which suggest that the probability of balking is increased by decreasing inter-arrival skewness and increasing service skewness.

These contrasting results exhibit unexpected behavior because it is generally assumed that improving a mean performance measure will improve other performance measures. For example, improvements gained from reducing the variance of inter-arrival and service times are lower mean and variance of waiting times and lower coefficient of variation of inter-departure times. However, GI/G/- queues have previously exhibited unexpected behavior (Romero-Silva & Hurtado, 2017; Whitt, 1980, 1984; Wolff, 1977) that challenges standard queueing theory notions. Thus, some conjectures are provided here to attempt explain their divergence from the norm.

One possible reason for negatively skewed inter-arrival distributions reducing CV_{ID} is that with the higher probability of highly 'spaced' arrivals (longer inter-arrival times), departure regularity would depend only on the regularity of the service process and would not be the result of the interaction (and potential propagation effect) between the variabilities of inter-arrival and service times. Therefore, a combination of highly spaced arrivals with the probability of rather quick services (positive service skewness) could result in more regular departures.

Similar reasoning could apply to $Lag1_{ID}$. The same combination of negative γ_A and positive γ_S results in less negative $Lag1_{ID}$ values. This, consequently, results in less dependency between consecutive departures since the interaction between γ_A and γ_S could be minimal and the departures would only depend on the service process, which is not

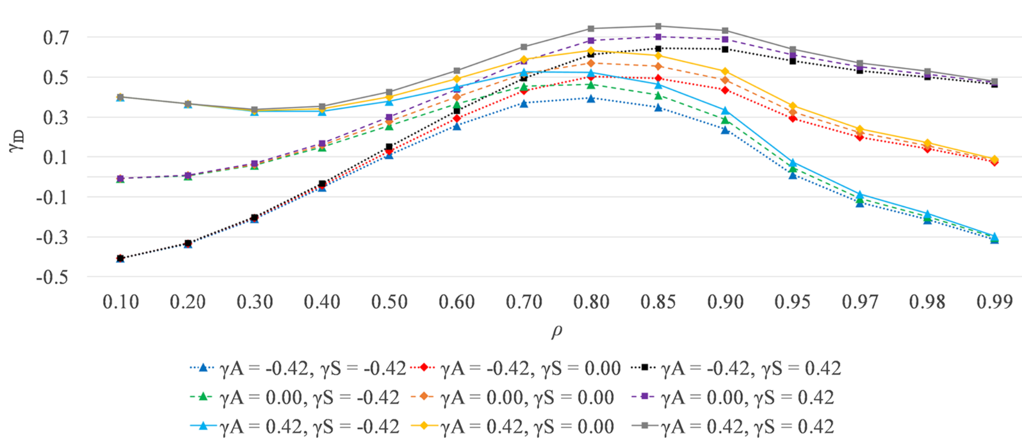

Fig. 6. γ_{ID} values for experiments with a system's CV = 0.3642.

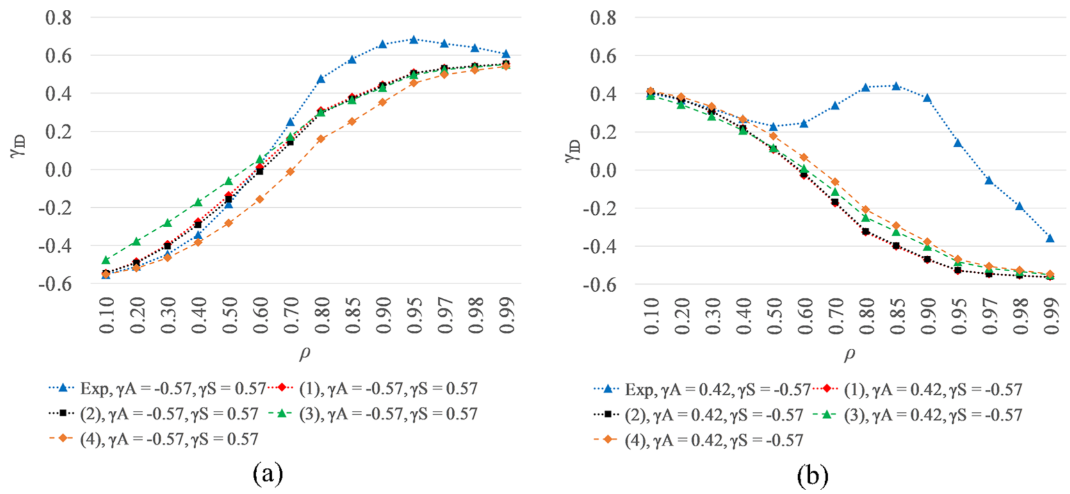

Fig. 7. Comparison of formulas to estimate γ_{ID} against experimental results for different values of ρ and CV = 0.2318 and selected contrasting values of γ_A and γ_S.

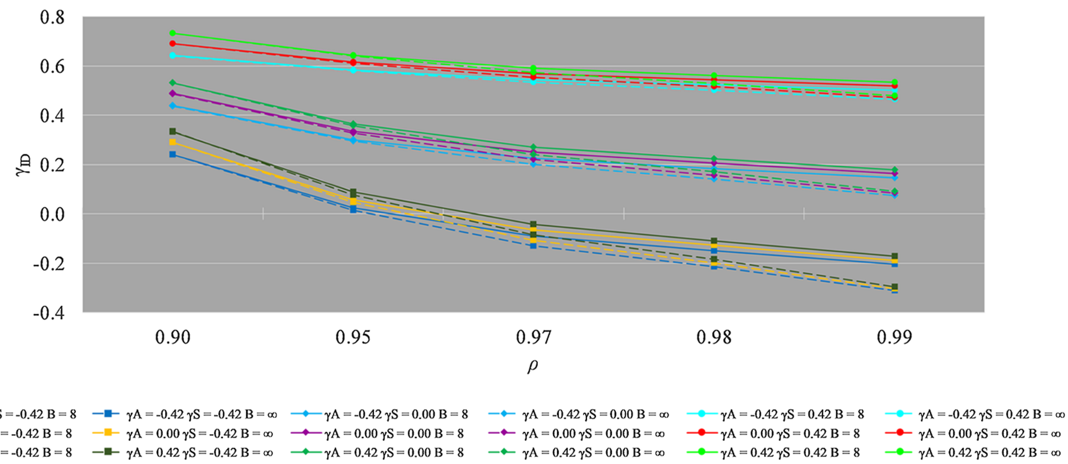

Fig. 8. Effect of γ_A and γ_S on γ_{ID} for different values of B for $\rho \geq 0.90$ and CV = 0.3642.

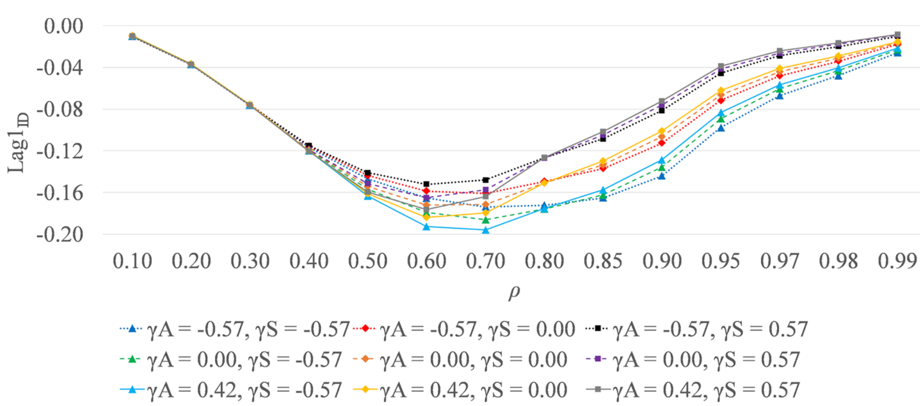

Fig. 9. $Lag1_{ID}$ values for experiments with CV = 0.2318.

auto-correlated. However, positive γ_A at higher values of ρ resulting in less negative $Lag1_{ID}$ values, remains unexpected. One possible explanation could be that at higher values of ρ, higher values of γ_A result in a lower probability of no wait (Romero-Silva et al., 2017), which means any arrival under these conditions will have a higher probability of waiting for service. The resultant higher probability of waiting for service then may have the same limiting effect on the values of $Lag1_{ID}$ than high values of ρ have on $Lag1_{ID}$.

In addition to contributions provided through a comprehensive skewness evaluation, this study contributes by comparing experimental results with formulas proposed by other authors. The comparison reveals that previous studies have mostly considered systems that are traffic-intensive (i.e., high levels of ρ and highly variable with a CV of nearly 1) for their estimations, since the estimation errors of these formulas were relatively high for intermediate values of ρ caused by the 'hump' behavior of CV_{ID} and γ_{ID} (see Figs. 3 and 7). This difference demonstrates the need for more research on queueing systems that are not directly associated with the special characteristics of the variability of the exponential distribution, i.e. CV = 1, γ = 2, especially as the actual value of the coefficient of variation of real tasks could be

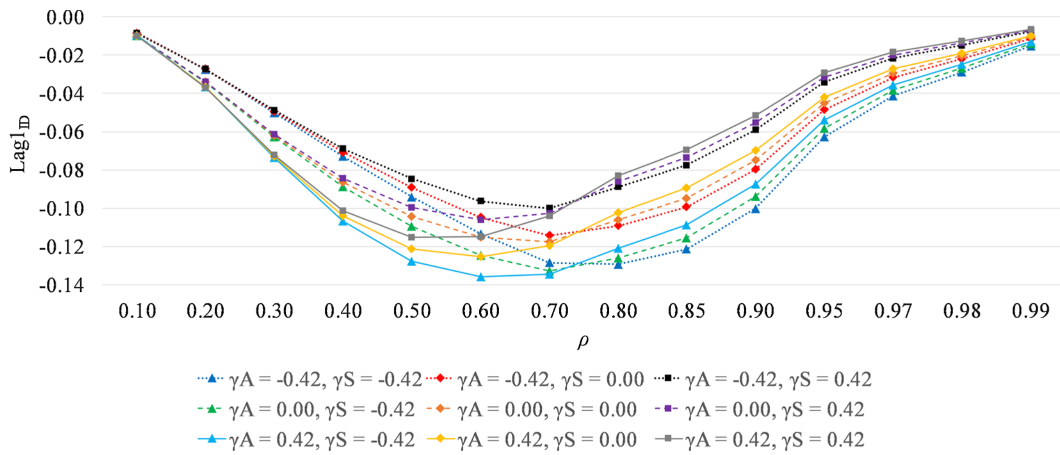

Fig. 10. Lag1_{ID} values for experiments with CV = 0.3642.

Table 4
ANOVA test for Lag1_{ID}.

	Sum Sq	F value	Pr(> F)
γ_A	0.020	8.080	0.004
γ_S	5.730	2281.916	0.000
ρ	0.220	89.200	0.000
CV	13.310	5301.035	0.000
B	0.030	6.696	0.001
γ_A:γ_S	0.000	0.945	0.331
γ_A:ρ	0.540	213.809	0.000
γ_S:ρ	2.330	925.941	0.000
γ_A:CV	0.070	29.224	0.000
γ_S:CV	0.000	0.487	0.485
ρ:CV	0.080	32.144	0.000
γ_A:B	0.000	0.003	0.997
γ_S:B	0.000	0.266	0.766
ρ:B	0.040	7.548	0.001
CV:B	0.010	2.601	0.074
γ_A:γ_S:ρ	0.010	2.375	0.123
γ_A:γ_S:CV	0.000	0.002	0.962
γ_A:ρ:CV	0.130	52.467	0.000
γ_S:ρ:CV	0.020	9.370	0.002
γ_A:γ_S:B	0.000	0.000	1.000
γ_A:ρ:B	0.000	0.004	0.996
γ_S:ρ:B	0.000	0.308	0.735
γ_A:CV:B	0.000	0.009	0.991
γ_S:CV:B	0.000	0.136	0.873
ρ:CV:B	0.010	2.850	0.058
γ_A:γ_S:ρ:CV	0.000	0.081	0.775
γ_A:γ_S:ρ:B	0.000	0.000	1.000
γ_A:γ_S:CV:B	0.000	0.001	0.999
γ_A:ρ:CV:B	0.000	0.010	0.990
γ_S:ρ:CV:B	0.000	0.154	0.857
γ_A:γ_S:ρ:CV:B	0.000	0.001	0.999
Residuals	189.740		

significantly lower than 1 (Doerr et al., 2004; Inman, 1999), which is more representative of the values used here.

Three well-recognized characteristics of stochastic queueing systems were present in these results. The limiting effect of ρ on the variability of system's output (BRAVO effect – Al-Hanbali et al., 2011; Nazarathy & Weiss, 2008) and the influence of the service time distribution on the performance of inter-departure times when the traffic intensity is high, were both seen, since the influence of γ_S on γ_{ID} as well as on Lag1_{ID} increased when ρ was high. Contrasting results were found regarding the effect of γ_A on γ_{ID}, which was diminished as ρ increased, a characteristic that had been reported by Johnson and Taaffe (1991). In addition, the impact of queue capacity on output variability found by previous studies (Hendricks & McClain, 1993; Kalir & Sarin, 2009) was also found in this study as lower values of B with high ρ resulted in both higher CV_{ID} and higher negative Lag1_{ID}; whereas B did not appear to have an influence on the effects caused by γ_A and γ_S across all the responses studied here.

Interestingly, the same graphical 'hump' found in values of CV_{ID} for intermediate values of ρ, is shown in the resulting values of γ_{ID}. An inverse hump can also be seen in values of Lag1_{ID} for intermediate values of ρ, independent of the values of γ_A and γ_S. This graphical inverse hump shows how the absolute magnitudes of Lag1_{ID} are highly influenced by ρ and presents some contrasting evidence to Yeh and Chang (2000), who suggested that dependency among departures increased with system's load. On the other hand, other results from this study confirm previous suggestions (Gerhardt & Nelson, 2010; Yeh & Chang, 2000) that dependency among inter-departure times is reduced by the variance of the service time distribution since the experiments with a CV = 0.3642 had lower negative autocorrelation values than experiments with a CV = 0.2318.

Note that the actual effect of decreasing values (negative) of Lag1_{ID} in downstream queues while considering serial lines could be difficult to assess, since various papers (Livny et al., 1993; Nielsen, 2007; Patuwo, Disney, & McNickle, 1993) have reported differing conclusions about the effect caused by negative inter-arrival correlation on single-server queues.

This overall 'hump' behavior of inter-departure times influenced by ρ could be the result of a propagation effect between the stochastic behavior of the distributions of inter-arrival and service times for intermediate values of ρ, resulting in an increased overall random behavior for the queue. If so, this would be contrary to the behavior of very low and very high traffic intensities where the behavior of the queue is only dependent on either the distribution of inter-arrival times, for very low traffic intensities, or on the distribution of service times, for very high traffic intensities.

The implications and contributions of this study are both theoretically valuable and expose gaps in current queueing theory understanding in the Operations Research field. They also address practical needs, since the inter-departure time distribution of a single station in a serial line is important to the performance of production lines, call-centers, hospitals and communication networks.

Improved control and management of γ_A and γ_S could create performance increases in serial lines, thus increasing competitive advantage by reducing γ_A and increasing γ_S. However, it may be challenging to implement an improvement process on the arrival and service processes that can directly influence the skewness of inter-arrival and service times, as most process improvement techniques are centered on variance reduction (George, Maxey, Rowlands, & Price, 2004; Santos, Wysk, & Torres, 2006). Moreover, the practical impacts of these findings regarding the reduction of inter-departure variance may be limited if an arrival process is not well-represented by probability distributions with negative skewness, e.g., triangular, beta, Johnson SB

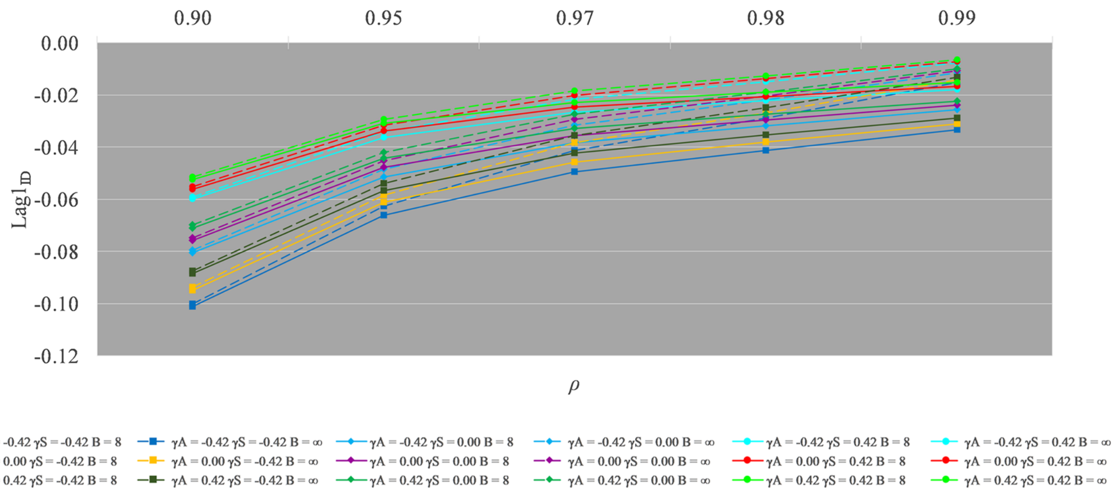

Fig. 11. Effect of γ_A and γ_S on $Lag1_{ID}$ for different values of B for $\rho \geq 0.90$ and CV = 0.3642.

and Johnson SU distributions.

These limitations and challenges generate interesting future research needs. One, for example, could be determining if the results of this study also hold for multiple-server queues and for inter-arrival and service time probability distributions other than the triangular, with different distribution 'shapes' and with CV values that are closer or higher than 1, since the conclusions of this study are limited by the use of the triangular distribution to model inter-arrival and service times, instead of the more commonly used theoretical distributions, e.g., exponential, hyper-exponential and Erlang; or more realistic distributions, e.g., Weibull (Dudley, 1963; Slack, 1982) and lognormal (Green, Kolesar, & Whitt, 2007; He, Liu, & Whitt, 2016).

Despite the fact that the current study considered two realistic queue capacity levels (one from an FTP packet-processing router and other from a commercial telephone device with a very limited capacity for holding external coming calls) more studies are needed to further investigate the interaction effects between the skewness and queue capacity for other values of queue capacity.

Further experimentation is also needed to determine if a direct reduction of inter-departure variance in a single-server queue through a reduction in the skewness of inter-arrival times could have higher line performance impact than the actual reduction of waiting times on a single-server queue by an increase in the skewness of inter-arrival times. Results from this study concerned with inter-arrival skewness show different conclusions than previous studies (Johnson, 1993; Whitt, 1984), meriting further research extension.

Comparing the impact of queue management policies, e.g., priority queueing (Romero-Silva, Shaaban, Marsillac, & Hurtado, 2018) and active queue management (Kim, Yoon, & Yeom, 2011), or the impact of lower input variance on the performance of single-server queues to the impact of inter-arrival and service skewness and queue capacity, could also shed light on potential improvements that can be attained by modifying the skewness, compared with policies and improvement processes that could be implemented in some queues.

Note that measuring the net impact of varying values of γ_A and γ_S on a serial line could prove difficult as this study has shown that γ_A and γ_S could have different effects on different characteristics of the inter-departure time distribution on an upstream station. These different effects could either improve or deteriorate the performance of downstream stations. Consequently, this issue presents challenges for authors who propose decomposition approximation techniques (Powell & Pyke, 1994; Whitt, 1994; Wu et al., 2018) to model the behavior of serial lines. Modelling all the characteristics of the departure process of one upstream queue to then model the arrival process of the next downstream queue could be a complex task.

6. Conclusions

This experimental study investigated (1) how the skewness of the distributions of inter-arrival and service times influenced the variability of inter-departure times and (2) whether a reduction on the variability of the departure process could be attained only by modifying the values of the skewness of inter-arrival and service times. To reach that objective, 756 experiments of a GI/G/1 queue with triangular distributions representing inter-arrival and service times were completed using discrete-event simulation. Five factors were included in the experimental design to analyze their impact on the variability of inter-departure times.

The variability of inter-departure times was shown to be influenced by the five experimental design factors. Furthermore, a combination of negative skewness of inter-arrival times and positive skewness of service times was found to reduce the coefficient of variation of inter-departure times, and vice versa. This same combination resulted in lower absolute values for the Lag-1 autocorrelation between consecutive departures for low to intermediate values of server's utilization.

Interestingly, all the main responses included in this investigation regarding the variability of inter-departure times, (i.e. the variance, coefficient of variation, skewness and Lag-1 autocorrelation), exhibited a 'hump' behavior characterized by an increase in their absolute values for intermediate values of the server's utilization, while their absolute values were reduced when the server's utilization was very low or very high, much like the BRAVO effect.

The practical implications of this study suggest that the random behavior of a system subject to stochastic processes is limited by high or low values of a server's utilization. This limitation would appear despite the detrimental impact on the mean waiting times that high values of a server's utilization will create. Furthermore, since the skewness of both inter-arrival and service times have varying effects on the characteristics of the departure process, practitioners and researchers should investigate the impact that any process improvement, such as Six Sigma, will produce on the variance of input distributions, and on the skewness of inter-arrival and service times.

Finally, study results suggest that further research is needed to investigate whether a reduction or an increase on the skewness of inter-arrival times should be applied to reduce the overall mean cycle time of the production line. Determining the correct process is needed since a reduction on the skewness of inter-arrival times reduces the inter-departure variability of a single-server queue but increases its mean waiting time, while an increase on the skewness of inter-arrival times increases the variability of inter-departure times but reduces the mean waiting time of such a queue, resulting in two very different performance metrics. Further research is also needed to assess whether these results hold for probability distributions other than the triangular distribution, which was used in this study.

Appendix A

See Fig. A1.
See Tables A1–A15.

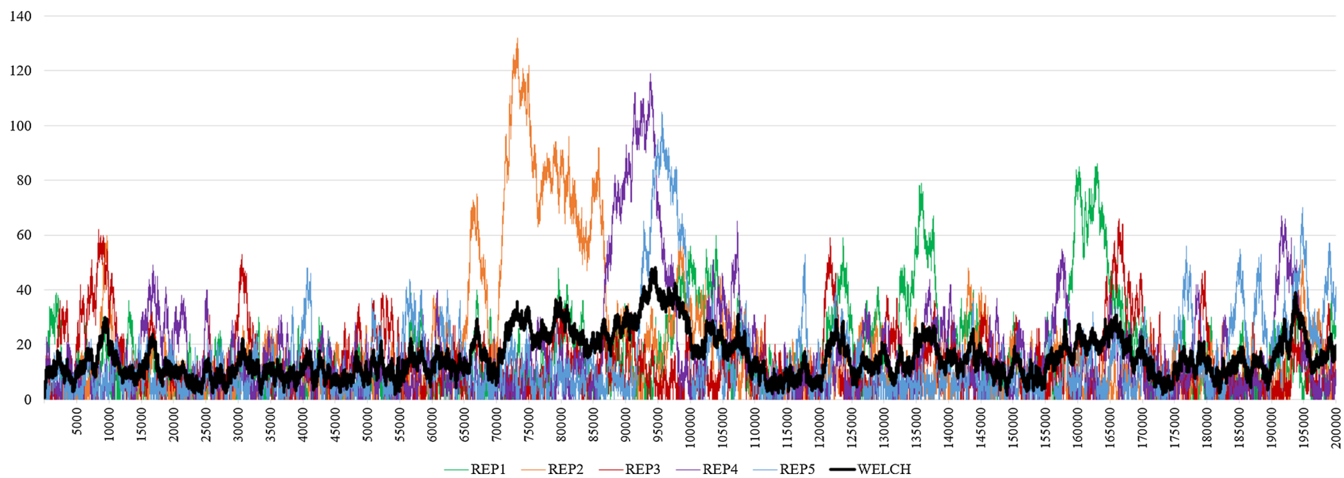

Fig. A1. Welch's method for a system with $\gamma_A = 0$, $\gamma_S = 0$, $\rho = 0.99$, 5 initial replications and a moving average considering 5 data points. REP*i* represents the *i*th replication in the Welch's exercise, while WELCH (black, solid line) represents the actual Welch procedure.

Table A1
Warm-up and run length depending on ρ values.

ρ	*w*	*n*
0.10	250	3000
0.20	250	3000
0.30	250	3000
0.40	250	3000
0.50	500	5000
0.60	500	5000
0.70	1000	10,000
0.80	1000	10,000
0.85	2000	20,000
0.90	2000	20,000
0.95	5000	100,000
0.97	5000	200,000
0.98	5000	200,000
0.99	7500	200,000

Table A2
Duncan's test groups showing statistically significant differences of average values of CV_{ID} among experiments with different skewness values with a CV = 0.2318. Each letter representing different groups with statistically significant differences.

		ρ													
γ_A	γ_S	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.85	0.90	0.95	0.97	0.98	0.99
−0.57	−0.57	a	a	a	b	f	g	g	g	f	e	d	d	d	c
−0.57	0.00	a	a	a	b	g	h	h	h	h	g	f	f	f	e
−0.57	0.57	a	a	a	b	h	i	i	i	i	h	g	g	g	f
0.00	−0.57	a	a	a	a	c	d	d	c	c	b	b	b	b	b
0.00	0.00	a	a	a	a	d	e	e	e	e	d	d	de	de	cd
0.00	0.57	a	a	a	a	e	f	f	f	g	f	e	f	f	e
0.42	−0.57	a	a	a	a	a	a	a	a	a	a	a	a	a	a
0.42	0.00	a	a	a	a	ab	b	b	b	b	b	c	c	c	b
0.42	0.57	a	a	a	a	b	c	c	d	d	c	d	e	e	d

Table A3
Duncan's test groups showing statistically significant differences of average values of CV_{ID} among experiments with different skewness values with a CV = 0.3642.

		ρ													
γ_A	γ_S	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.85	0.90	0.95	0.97	0.98	0.99
−0.42	−0.42	b	c	c	f	g	g	g	g	f	f	d	c	c	c
−0.42	0.00	b	c	c	fg	h	h	h	h	h	h	e	d	d	d
−0.42	0.42	b	c	c	g	i	i	i	i	i	i	f	e	e	e
0.00	−0.42	a	b	b	d	d	d	d	d	c	c	b	b	b	b
0.00	0.00	a	b	b	de	e	e	e	e	e	e	d	c	c	c
0.00	0.42	a	b	b	e	f	f	f	f	g	g	e	d	d	d
0.42	−0.42	a	a	a	a	a	a	a	a	a	a	a	a	a	a
0.42	0.00	a	a	a	b	b	b	b	b	b	b	b	b	b	b
0.42	0.42	a	a	a	c	c	c	c	c	d	d	c	c	c	c

Table A4
Duncan's test groups showing statistically significant differences of average values of CV_{ID} among experiments with different B values with a CV = 0.2318.

			ρ													
γ_A	γ_S	B	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.85	0.90	0.95	0.97	0.98	0.99
−0.57	−0.57	8	a	a	a	a	a	a	a	a	a	a	a	a	a	a
−0.57	−0.57	64	a	a	a	a	a	a	a	a	a	a	a	b	b	b
−0.57	−0.57	∞	a	a	a	a	a	a	a	a	a	a	a	b	b	b
−0.57	0.00	8	a	a	a	a	a	a	a	a	a	a	a	a	a	a
−0.57	0.00	64	a	a	a	a	a	a	a	a	a	a	a	b	b	b
−0.57	0.00	∞	a	a	a	a	a	a	a	a	a	a	a	b	b	b
−0.57	0.57	8	a	a	a	a	a	a	a	a	a	a	a	a	a	a
−0.57	0.57	64	a	a	a	a	a	a	a	a	a	a	a	b	b	b
−0.57	0.57	∞	a	a	a	a	a	a	a	a	a	a	a	b	b	b
0.00	−0.57	8	a	a	a	a	a	a	a	a	a	a	a	a	a	a
0.00	−0.57	64	a	a	a	a	a	a	a	a	a	a	a	a	b	b
0.00	−0.57	∞	a	a	a	a	a	a	a	a	a	a	a	a	b	b
0.00	0.00	8	a	a	a	a	a	a	a	a	a	a	a	a	a	a
0.00	0.00	64	a	a	a	a	a	a	a	a	a	a	a	b	b	b
0.00	0.00	∞	a	a	a	a	a	a	a	a	a	a	a	b	b	b
0.00	0.57	8	a	a	a	a	a	a	a	a	a	a	a	a	a	a
0.00	0.57	64	a	a	a	a	a	a	a	a	a	a	a	b	b	b
0.00	0.57	∞	a	a	a	a	a	a	a	a	a	a	a	b	b	b
0.42	−0.57	8	a	a	a	a	a	a	a	a	a	a	a	a	a	a
0.42	−0.57	64	a	a	a	a	a	a	a	a	a	a	a	a	b	b
0.42	−0.57	∞	a	a	a	a	a	a	a	a	a	a	a	a	b	b
0.42	0.00	8	a	a	a	a	a	a	a	a	a	a	a	a	a	a
0.42	0.00	64	a	a	a	a	a	a	a	a	a	a	a	b	b	b
0.42	0.00	∞	a	a	a	a	a	a	a	a	a	a	a	b	b	b
0.42	0.57	8	a	a	a	a	a	a	a	a	a	a	a	a	a	a
0.42	0.57	64	a	a	a	a	a	a	a	a	a	a	a	b	b	b
0.42	0.57	∞	a	a	a	a	a	a	a	a	a	a	a	b	b	b

Table A5
Duncan's test groups showing statistically significant differences of average values of CV_{ID} among experiments with different B values with a CV = 0.3642.

			ρ													
γ_A	γ_S	B	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.85	0.90	0.95	0.97	0.98	0.99
−0.42	−0.42	8	a	a	a	a	a	a	a	a	a	a	a	a	a	a
−0.42	−0.42	64	a	a	a	a	a	a	a	a	a	a	b	b	b	b
−0.42	−0.42	∞	a	a	a	a	a	a	a	a	a	a	b	b	b	b
−0.42	0.00	8	a	a	a	a	a	a	a	a	a	a	a	a	a	a
−0.42	0.00	64	a	a	a	a	a	a	a	a	a	a	b	b	b	b
−0.42	0.00	∞	a	a	a	a	a	a	a	a	a	a	b	b	b	b
−0.42	0.42	8	a	a	a	a	a	a	a	a	a	a	a	a	a	a
−0.42	0.42	64	a	a	a	a	a	a	a	a	a	a	b	b	b	b
−0.42	0.42	∞	a	a	a	a	a	a	a	a	a	a	b	b	b	b
0.00	−0.42	8	a	a	a	a	a	a	a	a	a	a	a	a	a	a
0.00	−0.42	64	a	a	a	a	a	a	a	a	a	a	b	b	b	b
0.00	−0.42	∞	a	a	a	a	a	a	a	a	a	a	b	b	b	b
0.00	0.00	8	a	a	a	a	a	a	a	a	a	a	a	a	a	a
0.00	0.00	64	a	a	a	a	a	a	a	a	a	a	b	b	b	b
0.00	0.00	∞	a	a	a	a	a	a	a	a	a	a	b	b	b	b
0.00	0.42	8	a	a	a	a	a	a	a	a	a	a	a	a	a	a
0.00	0.42	64	a	a	a	a	a	a	a	a	a	a	b	b	b	b

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Table A5 (*continued*)

γ_A	γ_S	B	ρ 0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.85	0.90	0.95	0.97	0.98	0.99
0.00	0.42	∞	a	a	a	a	a	a	a	a	a	a	b	b	b	b
0.42	−0.42	8	a	a	a	a	a	a	a	a	a	a	a	a	a	a
0.42	−0.42	64	a	a	a	a	a	a	a	a	a	a	b	b	b	b
0.42	−0.42	∞	a	a	a	a	a	a	a	a	a	a	b	b	b	b
0.42	0.00	8	a	a	a	a	a	a	a	a	a	a	a	a	a	a
0.42	0.00	64	a	a	a	a	a	a	a	a	a	a	b	b	b	b
0.42	0.00	∞	a	a	a	a	a	a	a	a	a	a	b	b	b	b
0.42	0.42	8	a	a	a	a	a	a	a	a	a	a	a	a	a	a
0.42	0.42	64	a	a	a	a	a	a	a	a	a	a	b	b	b	b
0.42	0.42	∞	a	a	a	a	a	a	a	a	a	a	b	b	b	b

Table A6
Duncan's test groups showing statistically significant differences of average values of γ_{ID} among experiments with different skewness values with a CV = 0.2318.

γ_A	γ_S	ρ 0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.85	0.90	0.95	0.97	0.98	0.99
−0.57	−0.57	c	c	c	d	i	i	i	h	g	i	i	i	i	i
−0.57	0.00	c	c	c	cd	h	h	h	g	e	f	f	f	f	f
−0.57	0.57	c	c	c	c	g	g	g	d	c	c	c	c	c	c
0.00	−0.57	b	b	b	b	f	f	f	g	f	h	h	h	h	h
0.00	0.00	b	b	b	b	e	e	e	e	d	e	e	e	e	e
0.00	0.57	b	b	b	b	d	d	c	b	b	b	b	b	b	b
0.42	−0.57	a	a	a	a	c	c	d	f	e	g	g	g	g	g
0.42	0.00	a	a	a	a	b	b	b	c	c	d	d	d	d	d
0.42	0.57	a	a	a	a	a	a	a	a	a	a	a	a	a	a

Table A7
Duncan's test groups showing statistically significant differences of average values of γ_{ID} among experiments with different skewness values with a CV = 0.3642.

γ_A	γ_S	ρ 0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.85	0.90	0.95	0.97	0.98	0.99
−0.42	−0.42	c	c	c	i	i	i	i	i	i	i	i	i	i	i
−0.42	0.00	c	c	c	h	h	h	h	g	f	f	f	f	f	f
−0.42	0.42	c	c	c	g	g	g	f	d	c	c	c	c	c	c
0.00	−0.42	b	b	b	f	f	f	g	h	h	h	h	h	h	h
0.00	0.00	b	b	b	e	e	e	e	e	e	e	e	e	e	e
0.00	0.42	b	b	b	d	d	d	c	b	b	b	b	b	b	b
0.42	−0.42	a	a	a	c	c	c	d	f	g	g	g	g	g	g
0.42	0.00	a	a	a	b	b	b	b	c	d	d	d	d	d	d
0.42	0.42	a	a	a	a	a	a	a	a	a	a	a	a	a	a

Table A8
Duncan's test groups showing statistically significant differences of average values of γ_{ID} among experiments with different B values with a CV = 0.2318.

γ_A	γ_S	B	ρ 0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.85	0.90	0.95	0.97	0.98	0.99
−0.57	−0.57	8	a	a	a	a	a	a	a	a	a	a	a	a	a	a
−0.57	−0.57	64	a	a	a	a	a	a	a	a	a	a	a	b	b	b
−0.57	−0.57	∞	a	a	a	a	a	a	a	a	a	a	a	b	b	b
−0.57	0.00	8	a	a	a	a	a	a	a	a	a	a	a	a	a	a
−0.57	0.00	64	a	a	a	a	a	a	a	a	a	a	a	a	b	b
−0.57	0.00	∞	a	a	a	a	a	a	a	a	a	a	a	a	b	b
−0.57	0.57	8	a	a	a	a	a	a	a	a	a	a	a	a	a	a
−0.57	0.57	64	a	a	a	a	a	a	a	a	a	a	a	a	b	b
−0.57	0.57	∞	a	a	a	a	a	a	a	a	a	a	a	a	b	b
0.00	−0.57	8	a	a	a	a	a	a	a	a	a	a	a	a	a	a

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Table A8 (*continued*)

γ_A	γ_S	B	ρ 0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.85	0.90	0.95	0.97	0.98	0.99
0.00	−0.57	64	a	a	a	a	a	a	a	a	a	a	a	a	b	b
0.00	−0.57	∞	a	a	a	a	a	a	a	a	a	a	a	a	b	b
0.00	0.00	8	a	a	a	a	a	a	a	a	a	a	a	a	a	a
0.00	0.00	64	a	a	a	a	a	a	a	a	a	a	a	b	b	b
0.00	0.00	∞	a	a	a	a	a	a	a	a	a	a	a	b	b	b
0.00	0.57	8	a	a	a	a	a	a	a	a	a	a	a	a	a	a
0.00	0.57	64	a	a	a	a	a	a	a	a	a	a	a	a	b	b
0.00	0.57	∞	a	a	a	a	a	a	a	a	a	a	a	a	b	b
0.42	−0.57	8	a	a	a	a	a	a	a	a	a	a	a	a	a	a
0.42	−0.57	64	a	a	a	a	a	a	a	a	a	a	a	a	b	b
0.42	−0.57	∞	a	a	a	a	a	a	a	a	a	a	a	a	b	b
0.42	0.00	8	a	a	a	a	a	a	a	a	a	a	a	a	a	a
0.42	0.00	64	a	a	a	a	a	a	a	a	a	a	a	a	b	b
0.42	0.00	∞	a	a	a	a	a	a	a	a	a	a	a	a	b	b
0.42	0.57	8	a	a	a	a	a	a	a	a	a	a	a	a	a	a
0.42	0.57	64	a	a	a	a	a	a	a	a	a	a	a	a	b	b
0.42	0.57	∞	a	a	a	a	a	a	a	a	a	a	a	a	b	b

Table A9
Duncan's test groups showing statistically significant differences of average values of γ_{ID} among experiments with different B values with a CV = 0.3642.

γ_A	γ_S	B	ρ 0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.85	0.90	0.95	0.97	0.98	0.99
−0.42	−0.42	8	a	a	a	a	a	a	a	a	a	a	a	a	a	a
−0.42	−0.42	64	a	a	a	a	a	a	a	a	a	a	b	b	b	b
−0.42	−0.42	∞	a	a	a	a	a	a	a	a	a	a	b	b	b	b
−0.42	0.00	8	a	a	a	a	a	a	a	a	a	a	a	a	a	a
−0.42	0.00	64	a	a	a	a	a	a	a	a	a	a	b	b	b	b
−0.42	0.00	∞	a	a	a	a	a	a	a	a	a	a	b	b	b	b
−0.42	0.42	8	a	a	a	a	a	a	a	a	a	a	a	a	a	a
−0.42	0.42	64	a	a	a	a	a	a	a	a	a	a	a	b	b	b
−0.42	0.42	∞	a	a	a	a	a	a	a	a	a	a	a	b	b	b
0.00	−0.42	8	a	a	a	a	a	a	a	a	a	a	a	a	a	a
0.00	−0.42	64	a	a	a	a	a	a	a	a	a	a	b	b	b	b
0.00	−0.42	∞	a	a	a	a	a	a	a	a	a	a	b	b	b	b
0.00	0.00	8	a	a	a	a	a	a	a	a	a	a	a	a	a	a
0.00	0.00	64	a	a	a	a	a	a	a	a	a	a	b	b	b	b
0.00	0.00	∞	a	a	a	a	a	a	a	a	a	a	b	b	b	b
0.00	0.42	8	a	a	a	a	a	a	a	a	a	a	a	a	a	a
0.00	0.42	64	a	a	a	a	a	a	a	a	a	a	b	b	b	b
0.00	0.42	∞	a	a	a	a	a	a	a	a	a	a	b	b	b	b
0.42	−0.42	8	a	a	a	a	a	a	a	a	a	a	a	a	a	a
0.42	−0.42	64	a	a	a	a	a	a	a	a	a	a	b	b	b	b
0.42	−0.42	∞	a	a	a	a	a	a	a	a	a	a	b	b	b	b
0.42	0.00	8	a	a	a	a	a	a	a	a	a	a	a	a	a	a
0.42	0.00	64	a	a	a	a	a	a	a	a	a	a	b	b	b	b
0.42	0.00	∞	a	a	a	a	a	a	a	a	a	a	b	b	b	b
0.42	0.42	8	a	a	a	a	a	a	a	a	a	a	a	a	a	a
0.42	0.42	64	a	a	a	a	a	a	a	a	a	a	b	b	b	b
0.42	0.42	∞	a	a	a	a	a	a	a	a	a	a	b	b	b	b

Table A10
Duncan's test groups showing statistically significant differences of average values of $Lag1_{ID}$ among experiments with different skewness values with a CV = 0.2318.

γ_A	γ_S	ρ 0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.85	0.90	0.95	0.97	0.98	0.99
−0.57	−0.57	a	a	a	a	bc	c	d	c	i	i	i	i	i	i
−0.57	0.00	a	a	a	a	ab	b	c	b	f	f	f	f	f	f
−0.57	0.57	a	a	a	a	a	a	a	a	c	c	c	c	c	c
0.00	−0.57	a	a	a	a	ef	e	f	d	h	h	h	h	h	h
0.00	0.00	a	a	a	a	de	d	d	b	e	e	e	e	e	e
0.00	0.57	a	a	a	a	cd	c	b	a	b	b	b	b	b	b
0.42	−0.57	a	a	a	a	h	g	g	d	g	g	g	g	g	g
0.42	0.00	a	a	a	a	gh	f	e	b	d	d	d	d	d	d
0.42	0.57	a	a	a	a	fg	e	c	a	a	a	a	a	a	a

Table A11
Duncan's test groups showing statistically significant differences of average values of $Lag1_{ID}$ among experiments with different skewness values with a CV = 0.3642.

		ρ													
γ_A	γ_S	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.85	0.90	0.95	0.97	0.98	0.99
−0.42	−0.42	a	ab	a	a	c	c	e	i	i	i	i	i	i	h
−0.42	0.00	a	a	a	a	b	b	c	f	f	f	f	f	f	e
−0.42	0.42	a	a	a	a	a	a	a	c	c	c	c	c	c	b
0.00	−0.42	a	c	b	b	f	d	f	h	h	h	h	h	h	g
0.00	0.00	a	bc	b	b	e	c	d	e	e	e	e	e	e	d
0.00	0.42	a	bc	b	b	d	b	ab	b	b	b	b	b	b	ab
0.42	−0.42	a	c	c	c	i	e	f	g	g	g	g	g	g	f
0.42	0.00	a	c	c	c	h	d	d	d	d	d	d	d	d	c
0.42	0.42	a	c	c	c	g	c	b	a	a	a	a	a	a	a

Table A12
Duncan's test groups showing statistically significant differences of average values of $Lag1_{ID}$ among experiments with different B values with a CV = 0.2318.

			ρ													
γ_A	γ_S	B	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.85	0.90	0.95	0.97	0.98	0.99
−0.57	−0.57	8	a	a	a	a	a	a	a	a	a	a	a	b	b	b
−0.57	−0.57	64	a	a	a	a	a	a	a	a	a	a	a	a	a	a
−0.57	−0.57	∞	a	a	a	a	a	a	a	a	a	a	a	a	a	a
−0.57	0.00	8	a	a	a	a	a	a	a	a	a	a	a	a	b	b
−0.57	0.00	64	a	a	a	a	a	a	a	a	a	a	a	a	a	a
−0.57	0.00	∞	a	a	a	a	a	a	a	a	a	a	a	a	a	a
−0.57	0.57	8	a	a	a	a	a	a	a	a	a	a	a	a	b	b
−0.57	0.57	64	a	a	a	a	a	a	a	a	a	a	a	a	a	a
−0.57	0.57	∞	a	a	a	a	a	a	a	a	a	a	a	a	a	a
0.00	−0.57	8	a	a	a	a	a	a	a	a	a	a	a	a	b	b
0.00	−0.57	64	a	a	a	a	a	a	a	a	a	a	a	a	a	a
0.00	−0.57	∞	a	a	a	a	a	a	a	a	a	a	a	a	a	a
0.00	0.00	8	a	a	a	a	a	a	a	a	a	a	a	a	b	b
0.00	0.00	64	a	a	a	a	a	a	a	a	a	a	a	a	a	a
0.00	0.00	∞	a	a	a	a	a	a	a	a	a	a	a	a	a	a
0.00	0.57	8	a	a	a	a	a	a	a	a	a	a	a	a	b	b
0.00	0.57	64	a	a	a	a	a	a	a	a	a	a	a	a	a	a
0.00	0.57	∞	a	a	a	a	a	a	a	a	a	a	a	a	a	a
0.42	−0.57	8	a	a	a	a	a	a	a	a	a	a	a	a	b	b
0.42	−0.57	64	a	a	a	a	a	a	a	a	a	a	a	a	a	a
0.42	−0.57	∞	a	a	a	a	a	a	a	a	a	a	a	a	a	a
0.42	0.00	8	a	a	a	a	a	a	a	a	a	a	a	a	b	b
0.42	0.00	64	a	a	a	a	a	a	a	a	a	a	a	a	a	a
0.42	0.00	∞	a	a	a	a	a	a	a	a	a	a	a	a	a	a
0.42	0.57	8	a	a	a	a	a	a	a	a	a	a	a	a	b	b
0.42	0.57	64	a	a	a	a	a	a	a	a	a	a	a	a	a	a
0.42	0.57	∞	a	a	a	a	a	a	a	a	a	a	a	a	a	a

Table A13
Duncan's test groups showing statistically significant differences of average values of $Lag1_{ID}$ among experiments with different B values with a CV = 0.3642.

			ρ													
γ_A	γ_S	B	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.85	0.90	0.95	0.97	0.98	0.99
−0.42	−0.42	8	a	a	a	a	a	a	a	a	a	a	b	b	b	b
−0.42	−0.42	64	a	a	a	a	a	a	a	a	a	a	a	a	a	a
−0.42	−0.42	∞	a	a	a	a	a	a	a	a	a	a	a	a	a	a
−0.42	0.00	8	a	a	a	a	a	a	a	a	a	a	b	b	b	b
−0.42	0.00	64	a	a	a	a	a	a	a	a	a	a	a	a	a	a
−0.42	0.00	∞	a	a	a	a	a	a	a	a	a	a	a	a	a	a
−0.42	0.42	8	a	a	a	a	a	a	a	a	a	a	b	b	b	b
−0.42	0.42	64	a	a	a	a	a	a	a	a	a	a	a	a	a	a
−0.42	0.42	∞	a	a	a	a	a	a	a	a	a	a	a	a	a	a
0.00	−0.42	8	a	a	a	a	a	a	a	a	a	a	b	b	b	b
0.00	−0.42	64	a	a	a	a	a	a	a	a	a	a	a	a	a	a
0.00	−0.42	∞	a	a	a	a	a	a	a	a	a	a	a	a	a	a
0.00	0.00	8	a	a	a	a	a	a	a	a	a	a	b	b	b	b
0.00	0.00	64	a	a	a	a	a	a	a	a	a	a	a	a	a	a
0.00	0.00	∞	a	a	a	a	a	a	a	a	a	a	a	a	a	a

(continued on next page)

Table A13 (*continued*)

γ_A	γ_S	B	ρ 0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.85	0.90	0.95	0.97	0.98	0.99
0.00	0.42	8	a	a	a	a	a	a	a	a	a	a	b	b	b	b
0.00	0.42	64	a	a	a	a	a	a	a	a	a	a	a	a	a	a
0.00	0.42	∞	a	a	a	a	a	a	a	a	a	a	a	a	a	a
0.42	−0.42	8	a	a	a	a	a	a	a	a	a	a	b	b	b	b
0.42	−0.42	64	a	a	a	a	a	a	a	a	a	a	a	a	a	a
0.42	−0.42	∞	a	a	a	a	a	a	a	a	a	a	a	a	a	a
0.42	0.00	8	a	a	a	a	a	a	a	a	a	a	b	b	b	b
0.42	0.00	64	a	a	a	a	a	a	a	a	a	a	a	a	a	a
0.42	0.00	∞	a	a	a	a	a	a	a	a	a	a	a	a	a	a
0.42	0.42	8	a	a	a	a	a	a	a	a	a	a	b	b	b	b
0.42	0.42	64	a	a	a	a	a	a	a	a	a	a	a	a	a	a
0.42	0.42	∞	a	a	a	a	a	a	a	a	a	a	a	a	a	a

Table A14
Probability of balking for experiments with different γ_A, γ_S and B values with a CV = 0.2318.

γ_A	γ_S	B	ρ 0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.85	0.90	0.95	0.97	0.98	0.99
−0.57	−0.57	8	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00003	0.00036	0.00111	0.00300
−0.57	−0.57	64	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
−0.57	−0.57	∞	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
−0.57	0.00	8	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00003	0.00038	0.00114	0.00302
−0.57	0.00	64	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
−0.57	0.00	∞	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
−0.57	0.57	8	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00003	0.00040	0.00117	0.00304
−0.57	0.57	64	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
−0.57	0.57	∞	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
0.00	−0.57	8	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00002	0.00034	0.00108	0.00298
0.00	−0.57	64	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
0.00	−0.57	∞	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
0.00	0.00	8	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00003	0.00036	0.00110	0.00299
0.00	0.00	64	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
0.00	0.00	∞	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
0.00	0.57	8	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00003	0.00038	0.00113	0.00301
0.00	0.57	64	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
0.00	0.57	∞	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
0.42	−0.57	8	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00002	0.00032	0.00105	0.00295
0.42	−0.57	64	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
0.42	−0.57	∞	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
0.42	0.00	8	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00002	0.00034	0.00108	0.00297
0.42	0.00	64	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
0.42	0.00	∞	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
0.42	0.57	8	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00003	0.00036	0.00111	0.00300
0.42	0.57	64	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
0.42	0.57	∞	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000

Table A15
Probability of balking for experiments with different γ_A, γ_S and B values with a CV = 0.3642.

γ_A	γ_S	B	ρ 0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.85	0.90	0.95	0.97	0.98	0.99
−0.42	−0.42	8	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00001	0.00019	0.00239	0.00570	0.00837	0.01196
−0.42	−0.42	64	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00009
−0.42	−0.42	∞	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
−0.42	0.00	8	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00001	0.00021	0.00246	0.00577	0.00842	0.01200
−0.42	0.00	64	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00009
−0.42	0.00	∞	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
−0.42	0.42	8	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00001	0.00023	0.00255	0.00583	0.00848	0.01204
−0.42	0.42	64	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00009
−0.42	0.42	∞	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
0.00	−0.42	8	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00001	0.00016	0.00231	0.00562	0.00831	0.01194
0.00	−0.42	64	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00009
0.00	−0.42	∞	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000

(*continued on next page*)

Table A15 (*continued*)

			ρ													
γ_A	γ_S	B	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.85	0.90	0.95	0.97	0.98	0.99
0.00	0.00	8	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00001	0.00018	0.00238	0.00570	0.00836	0.01197
0.00	0.00	64	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00009
0.00	0.00	∞	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
0.00	0.42	8	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00001	0.00019	0.00244	0.00574	0.00842	0.01200
0.00	0.42	64	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00009
0.00	0.42	∞	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
0.42	−0.42	8	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00013	0.00222	0.00554	0.00826	0.01192
0.42	−0.42	64	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00009
0.42	−0.42	∞	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
0.42	0.00	8	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00014	0.00228	0.00560	0.00831	0.01195
0.42	0.00	64	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00009
0.42	0.00	∞	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
0.42	0.42	8	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00001	0.00017	0.00236	0.00566	0.00835	0.01198
0.42	0.42	64	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00009
0.42	0.42	∞	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000

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